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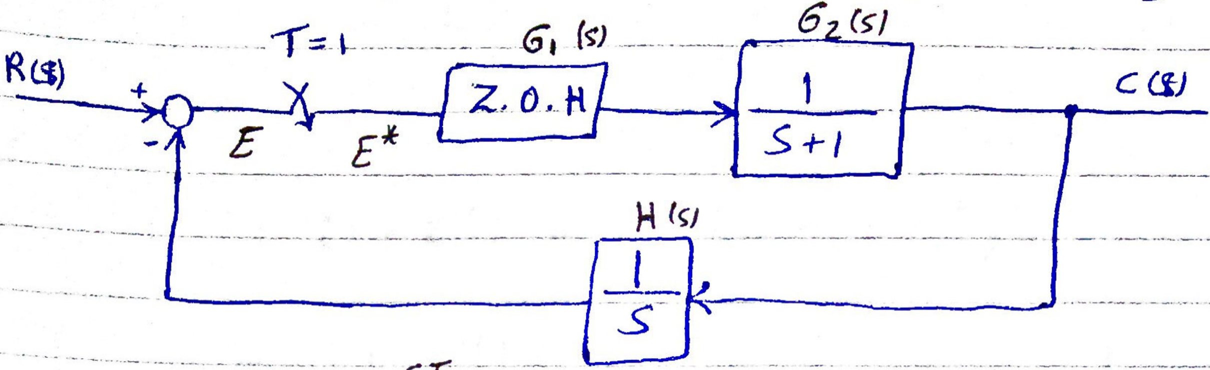
Sec 2

$\boxed{1}$ Find the unit-step response and discuss system stability



$$G_{Z.O.H}(S) = \frac{1-e^{-ST}}{S}$$

$$C(s) = E^* \, G_1 G_2(S) \xrightarrow{\text{Staring}} C^*(S) = E^* \, \overline{G_1 G_2}^*(S) \longrightarrow ①$$

$$E(S) = R(S) - C(S)\,H(S)$$

$$= R(S) - E^* \, G_1 G_2 H(S) \xrightarrow{\text{Starg}} E^*(s) = R^*(s) - E^*(s) \, \overline{G_1 G_2 H}^*(s) \rightarrow ②$$

from ② $\longrightarrow$ $E^*(s)\left(1 + \overline{G_1 G_2 H}^*(s)\right) = R^*(s)$ $\longrightarrow$ in ①

$$C^*(S) = \frac{R^*(s) \quad \overline{G_1 G_2}^*(s)}{1 + \overline{G_1 G_2 H}^*(s)}$$

$$C(Z) = \frac{R(Z) \quad \overline{G_1 G_2}(Z)}{1 + \overline{G_1 G_2 H}(Z)} \longrightarrow \boxed{3}$$

$$\overline{G_1 G_2}(z) = Z\left[\frac{1-e^{-ST}}{S} \cdot \frac{1}{S+1}\right] = (1-z^{-1})\,Z\left[\frac{1}{S(S+1)}\right]$$

$$= (1-z^{-1})\,Z\left[\frac{1}{S} + \frac{-1}{S+1}\right]$$

$$= (1-z^{-1})\,Z\left[u(t) - e^{-t}\right]$$

$$= (1-z^{-1})\left[\frac{Z}{Z-1} - \frac{Z}{Z-e^{-1}}\right] \longrightarrow$$

$$= 1 - \frac{Z-1}{Z-e^{-1}} = \frac{Z-e^{-1}-(Z-1)}{Z-e^{-1}} \qquad e^{-1} = 0.368$$

$$= \frac{0.632}{Z-0.368} \longrightarrow ④$$

$$\overline{G_1 G_2 H}(z) = Z\left[\frac{1-e^{-sT}}{s} \cdot \frac{1}{s+1} \cdot \frac{1}{s}\right] = (1-z^{-1}) Z\left[\frac{1}{s^2(s+1)}\right]$$

$$= (1-z^{-1})\, Z\left[\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}\right]$$

$$= (1-z^{-1})\, Z\left[t - u(t) + e^{-t}\right]$$

$$= (1-z^{-1})\left[\frac{z}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-1}}\right]$$

$$= \frac{1}{z-1} - 1 + \frac{z-1}{z-0.368}$$

$$= \frac{0.368\, z + 0.264}{(z-1)(z-0.368)} \longrightarrow 5$$

From 4, 5 in $\boxed{3}$

$$C(z) = R(z) \frac{\dfrac{0.632}{z-0.368}}{1 + \dfrac{0.368\, z + 0.264}{(z-1)(z-0.368)}}$$

$$= R(z) \frac{0.632\,(z-1)}{(z-1)(z-0.368) + 0.368\, z + 0.264}$$

$$= R(z) \frac{0.632\,(z-1)}{z^2 - 1.368 + 0.368 + 0.368\, z + 0.264}$$

$$= R(z) \frac{0.632\,(z-1)}{z^2 - z + .632}$$

unit step response

$$C(z) = \frac{z}{z-1} \cdot \frac{0.632\,(z-1)}{z^2 - z + 0.632} = \frac{0.632\, z}{z^2 - z + 0.632}$$

~~2 cos (w) = 1~~

~~cos (w) = 0.5~~

~~w = 60° = π/3 rad~~

~~sin (w) = 0.866~~

$$C(z) = \frac{0.632\, z}{z^2 - z + 0.632}$$

$$\sin(\omega t) \longrightarrow \frac{\sin(\omega)\, z}{z^2 - 2\cos(\omega)\, z + 1}$$

$$a^t \sin(\omega t) \longrightarrow \frac{\sin(\omega)\, z / a}{\frac{z^2}{a^2} - 2\cos(\omega)\frac{z}{a} + 1}$$

$$= \frac{a \sin(\omega)\, z}{z^2 - 2a\cos(\omega) z + a^2}$$

$$a^2 = 0.632 \longrightarrow a = 0.795$$

$$-2a\cos(\omega) = -1$$

$$\cos(\omega) = \frac{1}{2a} = 0.628$$

$$\omega = 51.0278$$

$$\sin(\omega) = 0.777$$

$$c(t) = (0.795)^t \sin(51.027^\circ t) \left[\frac{0.632}{0.795 * 0.777}\right]$$

$$= (1.023)\,(0.795)^t \sin(51.027^\circ t)$$

Study stability

$$S.S.e = R(t) - c(\infty)$$

$$= 1 - \lim_{z \to 1} c(z)\,(z-1)$$

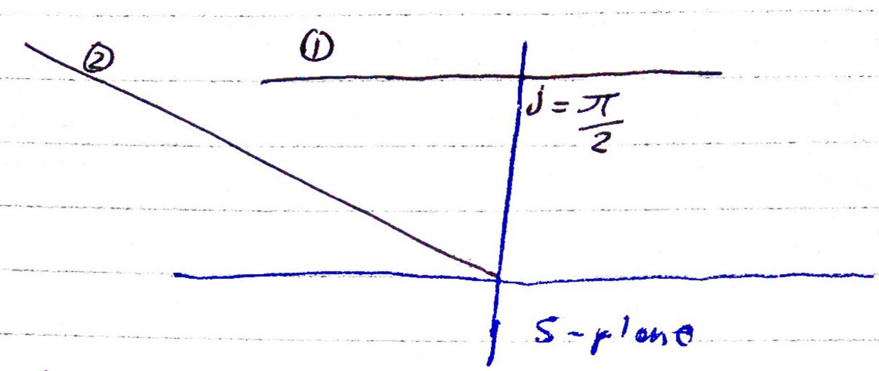
$$= 1 - \lim_{z \to 1} (z-1) \frac{0.632\, z}{z^2 - z + 0.632}$$

$$S.S.e = 1$$

There is an error equal 100 % of the desired output

So system is not stable

$\boxed{2}$ map to z-plane



For Line ①

→ σ changes from $-\infty$ to ∞

So $r = e^{\sigma}$ changes from 0 to ∞

→ w has a fixed value equal $\frac{\pi}{2} = 90°$

So $\theta = w = \frac{\pi}{2}$

We can represent that with line with angle $\frac{\pi}{2}$ start from 0 to ∞ in radius

For Line ②

→ Line lies in the left side

So $r < 0$

→ σ changes from 0 to $-\infty$

So r changes from 1 to 0

→ w changes from 0 to ∞

So θ changes from 0 to ∞

We can represent that with  that start with $r = 1$ with $\theta = 0$

and end with $r = 0$

angle become more positive

So curve move

to the positive direction angle